Q What is Regular Expression ? Explain closure properties of regular Language.

⇒ Regular Expression:

The language accepted by finite automata can be represented by some expression called as regular expression

Formal Definition:

The class of language over Σ can be defined recursively as:

1. ϕ (Null set) and ϵ (empty string) are always regular expression.
2. If $\Sigma = \{a\}$, then a is also regular expression.
3. If R_1 and R_2 are regular expression over Σ, then the combination ($R_1 + R_2$) (union), ($R_1 . R_2$) (series) and R_1 or R_2 (closure) are also regular expression.
4. Any expression formed using above points (1), (2) and (3) are also regular expression.

Closure Property:

Closure Properties on regular language are defined as certain operations on regular language. Which are guaranteed to produce regular language.

Closure refers to some operation on a language resulting in a new language that is of same "type" as originally operated on, i.e. regular.

5 In an automata Theory there are different closure property for regular language they are as follows.

b (i) Union: If L1 and L2 are 2 regular language. their Union L1 Union L2 will also be regular.

eg: $L_1 = \{a | n \geq 0\}$

$L_2 = \{b^n | n \geq 0\}$

$L_3 = L_1 \cup L_2 = \{a^n \cup b^n | n \geq 0\}$

(2) Intersection: If L1 and L2 are 2 regular language. Their $\cap$ $L_1 \cap L_2$ will be regular.

eg: $L_1 = \{a^m b^n | n \geq 0 \; m > 0\}$

$L_2 = \{a^m b^n \cup b^n a^m | n > 0, m > 0\}$

$L_3 = \{L_1 \cup L_2 = a^m b^n \; w | m > 0 \cap n > 0\}$ is also regular.

(3) Concatination: If L1 & L2 are 2 regular language then their concatination L1.L2 will also be regular

eg: $L_1 = \{a^m | m > 0\}$

$L_2 = \{b^n | n > 0\}$

$L_3 = L_1 . L_2 = \{a^m, b^n | m > 0, n > 0\}$ also regular

(4) Complement: ~~If L1 & L2 are 2 regular Language then th~~ If 6 L(G) is a regular is complement (L) will also be regular. Complement of Language can be found by subtracting a string which are in L(G) from all possible string.

$L(G) = \{a^n | n > 3\}$

$L'(G) = \{a^n | n \leq 3\}$

5] Kleen Closure:

If L1 is regular language than L1 closure also be regular eg:

$L_1 = (a \cup b)^*$

Two regular expression are equivalent if language generated by them are same.

Eg: $(a+b^*)^*$ and $(a+b)^*$ generate the same language. Every string which is generated and vice-verso.

Q] a] Write the regular expression for the language representing strings in which every 0 is immediatly followed by atleast 2 one's.

⇒

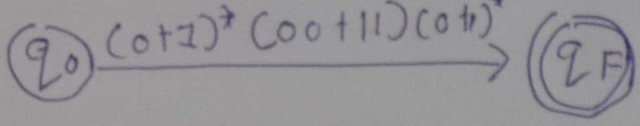
[?]

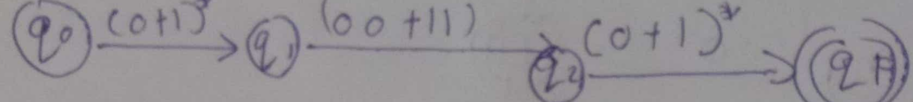
① Construct Regular r

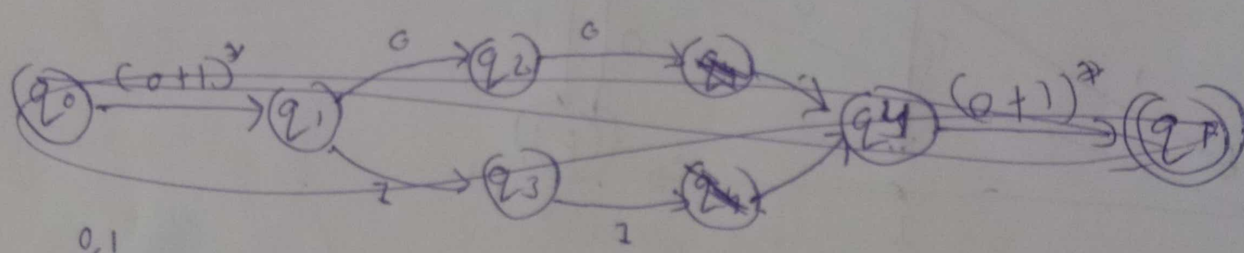
Q Convert the following regular expression to DFA.

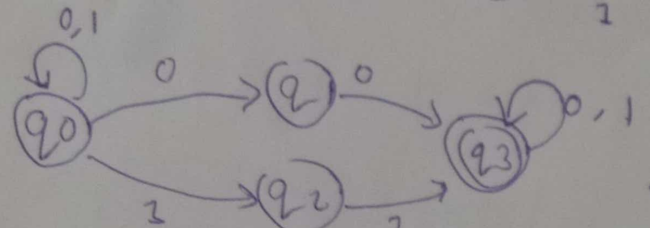
$(0+1)^* (00+11)(0+1)^*$.

⇒









⇒ NFA for given R.E

State table:

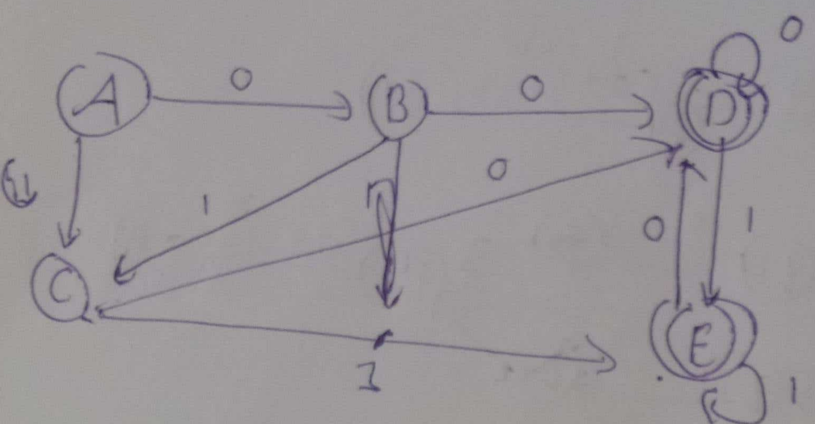
state	0	1
q_0	q_0, q_1	q_0, q_2
q_1	q_3	–
q_2	–	q_3
q_3	q_3	q_3

Transition

state	0	1
q_0	q_0, q_1	q_0, q_2
q_0, q_1	q_0, q_1, q_3	q_0, q_2
q_0, q_1, q_3	q_0, q_1, q_3	q_0, q_2, q_3
q_0, q_2, q_3	q_0, q_1, q_3	q_0, q_2, q_3
q_0, q_2	q_0, q_1, q_3	q_0, q_2, q_3

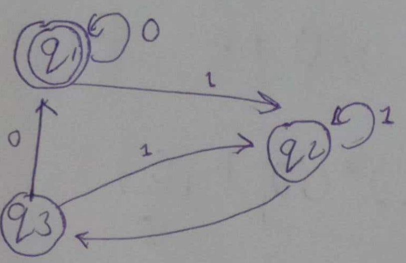
State	0	1
A	B	C
B	D	C
D	D	E
E	D	E
C	D	E

Doubt: Is it necessary to replace E with D.



t necessary to
E with D

Construct Regular Expression from given graph.



Note $R = Q + RP$
$R = QP^*$
According to Arden theorem

$\Rightarrow$ Equation for all state:

$q_1 = q_1 0 + q_3 0 + \epsilon$ ..... (1)

$q_2 = q_1 1 + q_2 1 + q_3 1$ ..... (2)

$q_3 = q_2 0$ ..... (3)

Put $\boxed{q_3 = q_2 0}$ in eq (2) and (1)

$q_1 = q_1 0 + q_2 0 . 0 + \epsilon$ ..... (4)

$\cancel{q_2 = q_1 1 + q_2 1 + q_2 0 1}$ | $q_2 = q_1 1 + q_2 1 + q_2 01$

$\underbrace{q_2}_{R} = \underbrace{q_1 1}_{Q} + \underbrace{q_2}_{R} \underbrace{(1 + 01)}_{P}$

By Arden's theorem.

$q_2 = q_1 1 (1 + 01)^*$ ..... (5)

$\cancel{q_1 = q_1 0 + q}$

From equation (4) and (5).

$q_1 = q_1 0 + q_1 1(1+01)^* 0 . 0 + \epsilon$ ... (4)

$\underset{R}{q_1} = \underset{R}{q_1} \underbrace{(0 + 1(1+01)^* 00)}_{P} + \underbrace{\epsilon}_{Q}$

By Arden's theorem

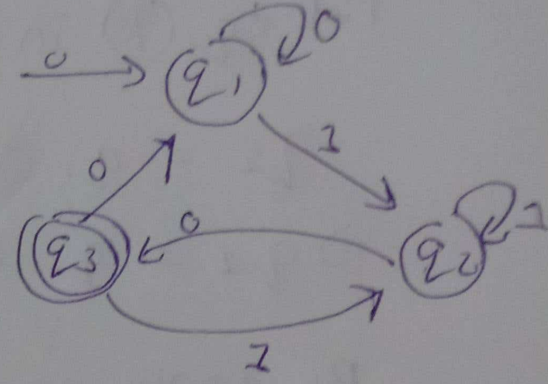
$q_1 = \epsilon\,(0 + 1(1+01)^* 00)^*$

∴ Regular Expression:

$q_1 = (0 + 1(1+01)^* 00)^*$

Using Pumping Lemma show ...

Q) Find regular expression



$\Rightarrow$ $q_0 = \epsilon$ ... (1)

$q_1 = q_0.0 + q_1.0 + q_3 0$ .. (2)

$q_2 = q_1 0 + q_2.1 + q_3.1$ .. (3)

$q_3 = q_2.0$ .. (4)

~~Put value of q_2 in eq (4)~~

~~$q_3 = (q_2.0 + q_2.1 + q_3.1).0$~~

~~$= q_2.00 + q_2.1\,0 + q_3.1\,0$.~~

Put eq (4) & (1) in (2) & (3)

$q_1 = q_1.0 + q_2.0.0 + \epsilon$ ... (5)

$q_2 = q_1 1 + q_2.1 + q_2.0.1$ ... (6)

In eq (6) by applying Arden theorem

$q_2 = q_1.1 + q_2.1 + q_2 0.1$

$\Rightarrow \underbrace{q_2}_{R} = \underbrace{q_1.1}_{Q} + \underbrace{q_2}_{R}\underbrace{[1 + 0.1]}_{P}$

$\boxed{\begin{array}{l} R = Q + RP \\ R = QP^* \end{array}}$

$\therefore q_2 = q_1.1.[1 + 0.1]^*$ ... (7)

Put eq (7) in eq (5)

$$q_2 = q_1 \cdot 0 + q_2 \cdot 0 \cdot 0.$$

$$\Rightarrow q_2 = q_1 \cdot 0 + [q_2 \cdot 1\,(1+0.1)^*] \cdot 0 \cdot 0 + \epsilon.$$

By applying Arden theorem.

$$\underbrace{q_2}_{R} = \underbrace{q_2}_{R} \underbrace{[0 + 1(1+0.1)^* \cdot 0 \cdot 0]}_{P} + \underbrace{\epsilon}_{Q}$$

$$q_2 = \epsilon\,[0+1\,(1+0)^* \cdot 0 \cdot 0]^* \quad \ldots (8)$$

put eq (8) in eq (7)

$$q_2 = \epsilon\,[0+1\,(1+0.1)^* \cdot 0 \cdot 0]^* \cdot 1\,[1+0]^* \quad \ldots (9)$$

Put eq (9) in eq (4).

$$q_3 = \epsilon\,[0+1\,(1+0.1)^* \cdot 0 \cdot 0]^* \cdot 1\,[1+0]^* \cdot 0$$

∴ The Regular expression is

$$\epsilon\,[0+1(1+0.1)^* \cdot 0 \cdot 0]^* \cdot 1\,[1+0]^* \cdot 0$$

Q) Using Pumping Lemma, show that following language are not Regular.

i) $L = \{0^i 1^i \mid i \geq 1\}$.

Step 1:

Let $i = 1$, length $= 0^1 1^1 = 01 = 2$

$i = 2$, length $= 0^2 1^2 = 0011 = 4$.

$i = 3$, length $= 0^3 1^3 = 000111 = 6$

Hence length of string in L is always even number.

Step 2:

Assume L is regular.

Let n is called as constant of Pumping Lemma.

Let $W \in L$ and $W = xyz$.

$\therefore W = 0^n 1^n$

$\therefore |W| = 2n = |xyz|$

Step 3: For $i = 1$

$xyz \in L$.

For $i = 2$, $\therefore 1 \leq |y| \leq n$.

$1 \leq |xy^2z| \leq n$.

But xy^2z is a combination of y and xyz.

Hence we will add the value of xyz ie. $|xyz| = 2n$.
on both side of equation (1)

$2n + 1 \leq |xy^2z| \leq n + 2n$.

$2n + 1 \leq |xy^2z| \leq 3n$

Subtrat 1 and Add 1.

$$2n < |xy^2z| < 3n+1.$$

For $n = 1$

$$2 < |xy^2z| < 4.$$

Length = 3 (Not even number)

Thus, the Language L consists of a strings whose length are not always even number.

So, our assumption that L is regular is wrong.

Hence by applying ~~pumgin~~ pumping lemma we prove that, L is not regular.

$\boxed{ii}$ $L = \{a^p / p \text{ is a prime}\}$ .... Pumping Lema,

$\Rightarrow$ Step 1:

Let $P = 2 = aa = 2$ (length)

$P = 3 = aaa = 3$ (-11-)

$P = 5 = aaaaa = 5$ (-11-)

Hence length of the string is always prime number

Step 2:

Assume L is regular.

Let n is called as constant of Pumping Lemma.

Let $W \in L$ and $W = XYZ$. $\therefore W = a^n$

$\therefore |W| = n = |xy^i z|$

Step 3:

For $\boxed{i=1}$

$xyz \in L$.

For $\boxed{i=2}$

$1 \leq y \leq n$

~~For~~ $1 \leq |xy^2 z| \leq n$

~~But~~ But xy^2z is the combination of y and xyz.

Hence we will add the value of xyz i.e.

$|xyz| = n$ on both side of equation()

$n+1 \leq |xy^2z| \leq n+n$

$n+1 \leq |xy^2z| \leq 2n$

Substract ~~and~~ 1 and add 1

$$n \le |xy^2z| \le 2n+1$$

put $\boxed{n=1}$

$$1 \le |xy^2z| \le \cancel{2n+1}\ 3$$

$$\therefore |xy^2z| = 2$$

Hence the length is 2 (prime)

put $\boxed{n=2}$

$$2 \le |xy^2z| \le 2(2)+1$$

$$2 \le |xy^2z| \le 5.$$

$$\therefore |xy^2z| = 3, 4$$

Hence the length is 3, 4 (not prime)

Hence the language L consists of a string whose length are not always prime.

Thus our assumption that L is regular is wrong.

~~Thus~~ Hence by applying pumping lemma we prove that, L is not regular.

Q $L = \{a^n b^{n+m} c^m \mid n, m \geq 1\}$ is not regular.

$\Rightarrow$ Let $n=1, m=1$, length (abbc) ~~length~~ $= 4$

$n=2, m=2$ length (aabbbbcc) $= 8$

$n=1, m=2$ length (abbbcc) $= 6$

Hence, the length of string above 4 is always even.

Now,
Assume L is regular language.
Let l be the constant of pumping lema.

~~Let W =~~

Let $W = a^l b^{l+l} c^l$ and $W = xyz$

$|w| = 4l = |xyz|$ $(l = 1, 2, 3, \ldots)$

Then, we have $1 \leq |y| \leq l$.

For $i=1$ $1 \leq |xyz| \leq l$.

For $i=2$ $1 \leq |xy^2z| \leq l$.

But xy^2z is the combination of both xyz and y.

$\therefore$ add xyz on both side.

$4l \leq |xy^2z| \leq l + 4l$.

$4l \leq |xy^2z| \leq 5l$.

Substract 1 and Add 1

$4l-1 \cancel{5l-1} < |xyz| \leq 5l+1$

Put $l=1$

$3 < |xyz| < 6.$

$|xyz|$ length is 4, 5 (odd)/(not even)

Put $l=2$

$7 < |xyz| < 11$

$\therefore |xyz|$ length is 8, 9, 10 where 9 is not even.

Hence for all length of the string is not even.
So, our assumption that L is regular is wrong.
Hence by pumping lemma L is not regular.